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Senior School Certificate Examination

March — 2010

Marking Scheme — Mathematics (Outside) 65/1, 65/2, 65/3

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. x   2. $\frac{2\pi}{3}$ 3. $x = 4$ 4. $-\frac{1}{4}\tan(7-4x)+c$ 5. $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$ 1x10 = 10 m

6. zero   7. 49   8. $\frac{1}{3}$ 9. $-6\hat{i}+3\hat{j}+6\hat{k}$ 10. -3

SECTION - B

11. Let event A is that the family has two boys

(i) event B: At least one is a boy

P(both boys, given that at least one is a boy) = P(A/B)   ½ m

$$= \frac{P(A\cap B)}{P(B)} = \frac{P\{(B, B)\}}{P\{(B, G), (G, B), (B, B),\}}$$ ½ + ½ m

$$= \frac{1/4}{3/4} = \frac{1}{3}$$ ½ m

(ii) event C: the elder child is a boy

P(both boys, given that at elder child is a boy) = P(A/C)

$$= \frac{P(A\cap C)}{P(C)} = \frac{P\{(B, B)\}}{P\{(B, G), (B, B),\}}$$ 1 m

$$= \frac{1/4}{2/4} = \frac{1}{2}$$ 1 m

12. (i) For all a ∈ A, (a, a) ∈ S ($\because$ a − a = 0 is divisible by 4)

$\therefore$ S is reflexive in A   1 m

(ii) For all a, b ∈ A, if (a, b) ∈ S then $|a-b|$ is divisible by 4.

Hence $|b-a|$ is also divisible by 4 ⇒ S is symmetric in A 1 m

(iii) ∀ a, b, c ∈ A, Let (a, b) ∈ S and (b, c) ∈ S

i.e. $|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

⇒ $(a-b) = \pm 4p$, $(b-c) = \pm 4q$, adding to get $a-c = 4m$ ⇒ (a, c) ∈ S 1½ m

⇒ S is transitive in A

Hence S is an equialence relation

Elements related to 1 are {1, 5, 9} ½ m

13. $\text{LHS} = \tan^{-1}\left[\dfrac{x+\dfrac{2x}{1-x^2}}{1-x\,\dfrac{2x}{1-x^2}}\right]$ 2 m

$= \tan^{-1}\left[\dfrac{x(1-x^2)+2x}{1-x^2-2x^2}\right]$ 1 m

$= \tan^{-1}\left[\dfrac{3x-x^3}{1-3x^2}\right] = \text{RHS.}$ 1 m

OR

$\text{LHS} = \cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

$= \cos\left[\tan^{-1}\left\{\sin\left(\sin^{-1}\dfrac{1}{\sqrt{1+x^2}}\right)\right\}\right]$ 1 m

$= \cos\left[\tan^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)\right] = \cos\left[\cos^{-1}\dfrac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right]$ 1+1 m

$= \dfrac{\sqrt{1+x^2}}{\sqrt{2+x^2}} = \text{R.H.S}$ 1 m

14. $A = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$, then $A' = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$ 1 m

Writing $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$ ½ m

$$\frac{1}{2}(A+A') = \begin{pmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{pmatrix}$$ ½ m

$$\frac{1}{2}(A-A') = \begin{pmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{pmatrix}$$ ½ m

and $\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{pmatrix}$ 1 m

Thus $A = B + C$

Where B is Symmetric matrix and C is skew symmetric matrix ½ m

15. $2\vec{a}-\vec{b}+3\vec{c} = (2\hat{i}+2\hat{j}+2\hat{k}) - (4\hat{i}-2\hat{j}+3\hat{k}) + (3\hat{i}-6\hat{j}+3\hat{k}) = \hat{i}-2\hat{j}+2\hat{k}$ 2 m

$|2\vec{a}-\vec{b}+3\vec{c}| = 3$ 1 m

$\therefore$ Required vector $= 2\hat{i}-4\hat{j}+4\hat{k}$ 1 m

OR

A vector perpendicutar to $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b} = 32\hat{i} - \hat{j} - 14\hat{k}$ ½+1 m

Let $\vec{d} = \lambda(32\hat{i} - \hat{j} - 14\hat{k})$ ½ m

$\therefore\ \vec{c} \cdot \vec{d} = 18 \Rightarrow \lambda(64+1-56) = 18 \Rightarrow \lambda = 2$ 1½ m

$\therefore\ \vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$ ½ m

16. Any point Q on the given line is Q $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ 1 m

$PQ^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 17\lambda^2 - 18\lambda - 16\lambda + 25$ 1 m

$PQ^2 = (5)^2 \Rightarrow 17\lambda(\lambda - 2) = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = 2$ 1 m

$\therefore$ The points are Q (–2, –1, 3) and R (4, 3, 7) ½+½ m

OR

Normal to the plane passing through A, B and C

is $\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 12\hat{i} - 16\hat{j} + 12\hat{k} \text{ or } 3\hat{i} - 4\hat{j} + 3\hat{k}$ 1½ m

$\therefore$ Equation of plane is $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19 \text{ or } 3x - 4y + 3z - 19 = 0$ 1½ m

Distance of P(6, 5, 9) from the plane $= \dfrac{|18 - 20 + 27 - 19|}{\sqrt{(3)^2 + (-4)^2 + (3)^2}}$

$= \dfrac{6}{\sqrt{34}}$ 1 m

17. Given differential equation can be written as

$\dfrac{dy}{dx} + \dfrac{2x}{x^2-1} \cdot y = \dfrac{1}{(x^2-1)^2}$ 1 m

Which is of the form $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

$$\int P(x)\,dx = \int \frac{2x}{x^2-1}\,dx = \log\left|x^2-1\right|$$ ½ m

$\therefore$ Integrating factor $= e^{\int P(x)\,dx} = (x^2-1)$ 1 m

$\therefore$ The solution is $(x^2-1)\cdot y = \int \frac{1}{(x^2-1)^2}(x^2-1)\,dx$ 1 m

$$(x^2-1)\cdot y = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$$ ½ m

OR

Given differential equation can be written as

$$\sqrt{(1+x^2)}\cdot\sqrt{(1+y^2)} + xy\frac{dy}{dx} = 0$$ ½ m

$$\Rightarrow \frac{y}{\sqrt{1+y^2}}\,dy = -\frac{\sqrt{1+x^2}}{x}\,dx$$ ½ m

Integrating both sides, we get

$$\sqrt{1+y^2} = -\int\frac{\sqrt{1+x^2}}{x^2}\cdot x\,dx = -\int\frac{t^2\,dt}{t^2-1} \text{ where } (1+x^2)=t^2$$ 1 m

$$\Rightarrow \sqrt{1+y^2} = -\int\left(1+\frac{1}{t^2-1}\right)dt = -t-\frac{1}{2}\log\frac{t-1}{t+1}+c$$

$$= -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$ 1+1 m

$$\text{or } \sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

18. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+2\,y/x}{1-y/x} = f\left(y/x\right)$$

hence, the differential equation is homogeneous. 1 m

Taking $\frac{y}{x} = v$ or $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$ ½ m

$$\therefore\ v + x\frac{dv}{dx} = \frac{1+2v}{1-v} \text{ or } x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \int \frac{v-1}{v^2+v+1}\,dv = -\int \frac{dx}{x}$$ 1 m

$$\Rightarrow \frac{1}{2}\int \frac{2v+1-3}{v^2+v+1}\,dv = -\log|x| + c$$

$$\text{or } \frac{1}{2}\log\left|v^2+v+1\right| - \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = -\log|x| + c$$

$$\Rightarrow \log\left|v^2+v+1\right| + \log x^2 = 2\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) + c$$ 1 m

$$\Rightarrow \log\left|y^2+xy+x^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + c$$ ½ m

19. Here $I = \int \frac{x+2}{\sqrt{x^2-5x+6}}\,dx = \frac{1}{2}\int \frac{2x-5+9}{\sqrt{x^2-5x+6}}\,dx$ 1 m

$$= \frac{1}{2}\int \frac{2x-5}{\sqrt{x^2-5x+6}}\,dx + \frac{9}{2}\int \frac{1}{\sqrt{\left(x-5/2\right)^2 - \left(\frac{1}{2}\right)^2}}\,dx$$ 1 m

$$= \sqrt{x^2-5x+6} + \frac{9}{2}\log\left|\left(x-\frac{5}{2}\right) + \sqrt{x^2-5x+6}\right| + c$$ 1+1 m

20. $I = \int_1^2 \frac{5x^2}{x^2+4x+3}\,dx = 5\int_1^2 1 - \frac{4x+3}{x^2+4x+3}\,dx$ 1 m

$= 5[x]_1^2 - 10\int_1^2 \frac{2x+4-\frac{5}{2}}{x^2+4x+3}\,dx$ ½ m

$= 5 - 10\left[\log|x^2+4x+3|\right]_1^2 + 25\int_1^2 \frac{1}{(x+2)^2-(1)^2}\,dx$ 1 m

$= 5 - 10\log\frac{15}{8} + 25\cdot\frac{1}{2}\left[\log\left|\frac{x+2-1}{x+2+1}\right|\right]_1^2$ 1 m

$= 5 - 10\log\frac{15}{8} + \frac{25}{2}\log\frac{6}{5}$ ½ m

Note: If solved using partial fractions, the answer be of the form

$$5 + \frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\left(\frac{5}{4}\right)$$

21. $\frac{dy}{dx} = e^{a\sin^{-1}x}\,\frac{a}{\sqrt{1-x^2}} = \frac{ay}{\sqrt{1-x^2}}$ 1 m

$\Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = ay$ .... (i) ½ m

$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\cdot\frac{dy}{dx} = a\frac{dy}{dx}$ 1 m

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a\sqrt{1-x^2}\frac{dy}{dx} = 0$ ½ m

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$ [Using (i)] 1 m

22. $y = \cos^{-1}\left[\frac{3}{5}x + \frac{4}{5}\sqrt{1-x^2}\right]$

$= \cos^{-1}\left[\frac{3}{5}\cdot\cos\theta + \frac{4}{5}\sin\theta\right]$ where $x = \cos\theta$ 1 m

$= \cos^{-1}[\cos\alpha\cdot\cos\theta + \sin\alpha\cdot\sin\theta]$, $\because$ if $\frac{3}{5} = \cos\alpha$, then $\frac{4}{5} = \sin\alpha$ 1 m

$= \cos^{-1}[\cos(\alpha-\theta)] = \alpha-\theta = \cos^{-1}\left(3/5\right) - \cos^{-1}x$ 1 m

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\left[\text{Note: Answer can be } -\frac{1}{\sqrt{1-x^2}}\right]$ 1 m

SECTION - C

23. LHS $= \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + p\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$ 1 m

$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ ½ m

$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ ½ m

$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$ $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$ 1 m

$$= (1+pxyz)\,(x-y)\,(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & -1 & -(x+y) \\ 0 & 1 & z+x \end{vmatrix}$$ ½ m

$$= (1+pxyz)\,(x-y)\,(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 0 & z-y \\ 0 & 1 & z+x \end{vmatrix} \; R_2 \to R_2 + R_3$$ 1 m

$$= (1+pxyz)\,(x-y)\,(y-z)\,(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 0 & -1 \\ 0 & 1 & z+x \end{vmatrix}$$ ½ m

$$= (1+pxyz)\,(x-y)\,(y-z)\,(z-x)\cdot 1 = \text{R.H.S.}$$ 1 m

OR

Writing $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_2 \to R_2 + R_1 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_3 \to R_3 + 2R_2 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$ 1 m

$$R_1 \to R_1 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 10 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ ½ m

$$R_1 \to R_1 - 2R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ ½ m

$$\text{Hence } A^{-1} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$ 1 m

24. E_1 : Bag contains 2 white balls and 2 non whites

E_2 : Bag contains 3 white balls and 1 non whites 1 m

E_3 : Bag contains 4 white balls

A : Getting two white balls

$P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{3}$, $P(E_3) = \frac{1}{3}$ ½ m

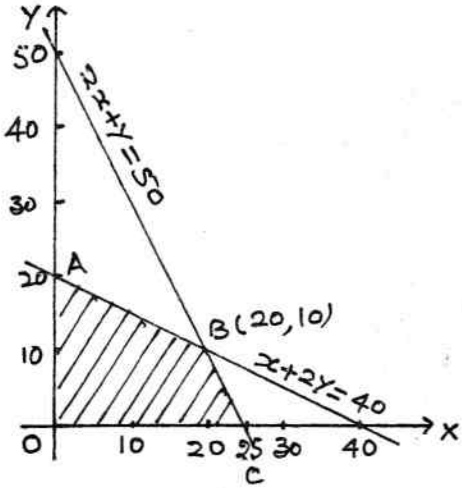
$P(A/E_1) = \frac{2c_2}{4c_2} = \frac{1}{6}$, $P(A/E_2) = \frac{3c_2}{4c_2} = \frac{1}{2}$, $P(A/E_3) = 1$ 1½ m

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) + P(E_3)\ P(A/E_3)}$$ 1 m

$$= \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1}$$ 1 m

$$= \frac{6}{10} = \frac{3}{5}$$ 1 m

25. Let x cakes of first type and y cakes of second type are made



Maximise $S = x + y$ 1 m

subject to $300x + 150y \le 7500$ or $2x + y \le 50$

$15x + 30y \le 600$ or $x + 2y \le 40$ 2 m

$x \ge 0, y \ge 0$

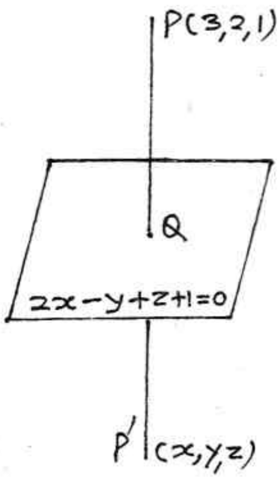
Correct graph 2 m

Vertices of feasible region are

A (0, 20), B (20, 10) C (25, 0)

Maximum cakes $= 20 + 10 = 30$ 1 m

26. Let Q be the foot of perpendicular from P to the plane



$\therefore$ Equation of PQ is $\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ 1 m

Any point on this line is $(2\lambda + 3, -\lambda + 2, \lambda + 1)$ ½ m

If this point is Q, then it must satisfy the equation of plane

$\therefore\ 2(2\lambda + 3) - (-\lambda + 2) + (\lambda + 1) + 1 = 0$ 1 m

$\Rightarrow\ \lambda = -1$ ½ m

$\therefore$ coordinates of foot of perpendiculare are Q (1, 3, 0) 1 m

Perpendicular distance = $PQ = \sqrt{4+1+1} = \sqrt{6}$ units 1 m

Let $P'(x, y, z)$ be the image, then $\frac{x+3}{2} = 1, \frac{y+2}{2} = 3, \frac{z+1}{2} = 0$ ½ m

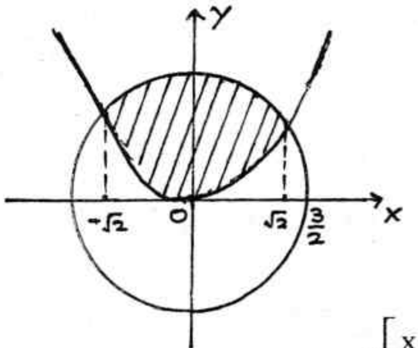
$\therefore\ P'$ is $(-1, 4, -1)$ 1 m

27. Correct Figure 1 m

Solving $4x^2 + 4y^2 = 9$ and $x^2 = 4y$

We get $x = \pm\sqrt{2}$ (as points of intersection)

or $y = \frac{1}{2}$ ½ m



Required area

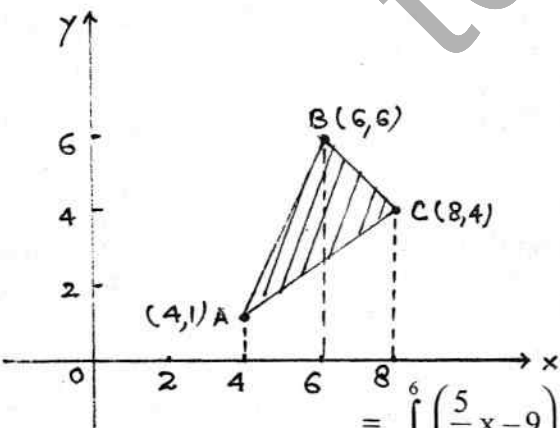
$$= 2\left[\int_0^{\sqrt{2}} \sqrt{\frac{9}{4} - x^2}\, dx - \int_0^{\sqrt{2}} \frac{1}{4} x^2\, dx\right]$$ 2 m

$$= 2\left[\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3} - \frac{x^3}{12}\right]_0^{\sqrt{2}}$$ 1½ m

$$= 2\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{2\sqrt{2}}{12}\right)$$ ½ m

$$= \left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right) \text{ sq. units}$$ ½ m

OR



Equations of AB, BC and AC respectively are

$y = \frac{5}{2}x - 9,\ y = 12 - x,\ y = \frac{3}{4}x - 2$ 1½ m

Required area

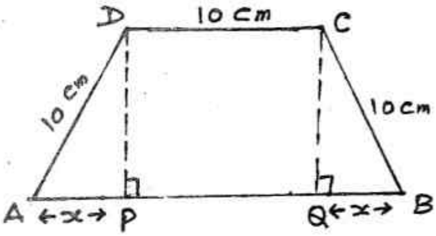
$$= \int_4^6 \left(\frac{5}{2}x - 9\right) dx + \int_6^8 (12 - x)\, dx - \int_4^8 \left(\frac{3}{4}x - 2\right) dx$$ 2 m

$$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$ 1½ m

$= (7 + 10 - 10)$ sq units 1 m

$= 7$ sq units

28. Let ABCD be the given trapezium with AD = DC = BC = 10 cm. Draw DP ⊥ AB and CQ ⊥ AB and let AP = x cm => QB = x cm 1 m



$\therefore$ Area of trapezium, $A = \frac{1}{2}\left[10 + (10 + 2x)\right]\sqrt{100 - x^2}$ 1 m

$$A = (x + 10)\sqrt{100 - x^2}$$

Let $S = (x + 10)^2 . (100 - x^2) \Rightarrow \frac{ds}{dx} = -2x\,(x + 10)^2 + 2\,(x + 10)\,(100 - x^2)$

$= 2\,(x + 10)^2 \cdot (-x + 10 - x)$ 1 m

$= 2\,(x + 10)^2 \cdot (10 - 2x)$

$\frac{ds}{dx} = 0 \Rightarrow x = 5$ [rejecting $x = -10$] 1 m

$\frac{d^2s}{dx^2} = -4\,(x + 10)^2 + 4\,(x + 10)\;(10 - 2x) = -900\;(-\text{ve})$ 1 m

$\therefore$ Maximum Area $A = 15\sqrt{75}\ \text{cm}^2$ *or* $75\sqrt{3}\ \text{cm}^2$ 1 m

29. Full marks to be given to every candidate for this question. 6 m

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. -3 2. zero 3. $-\frac{1}{4}\tan(7-4x)+c$ 4. $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$ 5. x — 1x10 = 10 m

6. $\frac{5\pi}{6}$ 7. $x=4$ 8. 49 9. $\frac{1}{3}$ 10. $\frac{\pi}{4}$

SECTION - B

11. LHS $= \tan^{-1}\left(\dfrac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\cdot\frac{1}{5}}\right) + \tan^{-1}\left(\dfrac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}\right)$ — 1½ m

$= \tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\left(\frac{3}{11}\right)$ — 1 m

$= \tan^{-1}\left(\dfrac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\cdot\frac{3}{11}}\right)$ — ½ m

$= \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS}$ — 1 m

OR

$= \tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\cdot\left(\frac{x+1}{x+2}\right)}\right) = \frac{\pi}{4}$ — 1½ m

$$\Rightarrow \frac{(x-1)(x+2)+(x-2)(x+1)}{(x^2-4)-(x^2-1)} = 1$$ 1 m

$$\Rightarrow x^2+x-2+x^2-x-2=-3$$ 1 m

$$\Rightarrow 2x^2-1=0 \quad \therefore x=\pm\frac{1}{\sqrt{2}}$$ ½ m

12. Let event A is that the family has two boys

(i) event B: At least one is a boy

P(both boys, given that at least one is a boy) = P(A/B) ½ m

$$=\frac{P(A\cap B)}{P(B)}=\frac{P\{(B, B)\}}{P\{(B, G), (G, B), (B, B),\}}$$ ½ + ½ m

$$=\frac{1/4}{3/4}=\frac{1}{3}$$ ½ m

(ii) event C: the elder child is a boy

P(both boys, given that at elder child is a boy) = P(A/C)

$$=\frac{P(A\cap C)}{P(C)}=\frac{P\{(B, B)\}}{P\{(B, G), (B, B),\}}$$ 1 m

$$=\frac{1/4}{2/4}=\frac{1}{2}$$ 1 m

13. (i) For all a ∈ A, (a, a) ∈ S (∵ a – a = 0 is divisible by 4)

∴ S is reflexive in A 1 m

(ii) For all a, b ∈ A, if (a, b) ∈ S then $|a-b|$ is divisible by 4.

Hence $|b-a|$ is also divisible by 4 ⇒ S is symmetric in A 1 m

(iii) $\forall$ a, b, c $\in$ A, Let (a, b) $\in$ S and (b, c) $\in$ S

i.e. $|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$\Rightarrow$ $(a-b) = \pm 4p$, $(b-c) = \pm 4q$, adding to get $a-c = 4m \Rightarrow (a, c) \in S$ 1½ m

$\Rightarrow$ S is transitive in A

Hence S is an equialence relation

Elements related to 1 are {1, 5, 9} ½ m

14. $A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$ 1 m

$A^2 - 3A + 2I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 1+1 m

$= \begin{pmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{pmatrix}$ 1 m

15. Any point Q on the given line is Q $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ 1 m

$PQ^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 17\lambda^2 - 18\lambda - 16\lambda + 25$ 1 m

$PQ^2 = (5)^2 \Rightarrow 17\lambda(\lambda - 2) = 0 \Rightarrow \lambda = 0$ or $\lambda = 2$ 1 m

$\therefore$ The points are Q (–2, –1, 3) and R (4, 3, 7) ½+½ m

OR

Normal to the plane passing through A, B and C

is $\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 12\hat{i} - 16\hat{j} + 12\hat{k}$ or $3\hat{i} - 4\hat{j} + 3\hat{k}$ 1½ m

$\therefore$ Equation of plane is $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$ or $3x - 4y + 3z - 19 = 0$ 1½ m

Distance of P(6, 5, 9) from the plane $= \dfrac{|18 - 20 + 27 - 19|}{\sqrt{(3)^2 + (-4)^2 + (3)^2}}$

$= \dfrac{6}{\sqrt{34}}$ 1 m

16. $2\vec{a} - \vec{b} + 3\vec{c} = (2\hat{i} + 2\hat{j} + 2\hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + (3\hat{i} - 6\hat{j} + 3\hat{k}) = \hat{i} - 2\hat{j} + 2\hat{k}$ 2 m

$|2\vec{a} - \vec{b} + 3\vec{c}| = 3$ 1 m

$\therefore$ Required vector $= 2\hat{i} - 4\hat{j} + 4\hat{k}$ 1 m

OR

A vector perpendicular to $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b} = 32\hat{i} - \hat{j} - 14\hat{k}$ ½+1 m

Let $\vec{d} = \lambda(32\hat{i} - \hat{j} - 14\hat{k})$ ½ m

$\therefore \vec{c} \cdot \vec{d} = 18 \Rightarrow \lambda(64 + 1 - 56) = 18 \Rightarrow \lambda = 2$ 1½ m

$\therefore \vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$ ½ m

17. Given differential equation can be written as

$\dfrac{dy}{dx} + \dfrac{2x}{x^2 - 1} \cdot y = \dfrac{1}{(x^2 - 1)^2}$ 1 m

Which is of the form $\dfrac{dy}{dx} + P(x) \cdot y = Q(x)$

$$\int P(x)\,dx = \int \frac{2x}{x^2-1}\,dx = \log\left|x^2-1\right|$$ ½ m

$\therefore$ Integrating factor $= e^{\int P(x)\,dx} = (x^2-1)$ 1 m

$\therefore$ The solution is $(x^2-1)\cdot y = \int \frac{1}{(x^2-1)^2}(x^2-1)\,dx$ 1 m

$$(x^2-1)\cdot y = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$$ ½ m

OR

Given differential equation can be written as

$$\sqrt{(1+x^2)}\cdot\sqrt{(1+y^2)} + xy\frac{dy}{dx} = 0$$ ½ m

$$\Rightarrow \frac{y}{\sqrt{1+y^2}}\,dy = -\frac{\sqrt{1+x^2}}{x}\,dx$$ ½ m

Integrating both sides, we get

$$\sqrt{1+y^2} = -\int\frac{\sqrt{1+x^2}}{x^2}\cdot x\,dx = -\int\frac{t^2\,dt}{t^2-1} \text{ where } 1+x^2=t^2$$ 1 m

$$\Rightarrow \sqrt{1+y^2} = -\int\left(1+\frac{1}{t^2-1}\right)dt = -t-\frac{1}{2}\log\frac{t-1}{t+1}\,c$$

1+1 m

$$= -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$

$$\text{or}\quad \sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

18. $$\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}\,dx$$ 1 m

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}\,dx - 7\int \frac{1}{\sqrt{(x+2)^2+\left(\sqrt{6}\right)^2}}\,dx$$ 1 m

$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + c$$ 1+1 m

19. $$\frac{dy}{dx} = e^{a\sin^{-1}x}\frac{a}{\sqrt{1-x^2}} = \frac{ay}{\sqrt{1-x^2}}$$ 1 m

$$\Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = ay \text{ .... (i)}$$ ½ m

$$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\cdot\frac{dy}{dx} = a\frac{dy}{dx}$$ 1 m

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a\sqrt{1-x^2}\frac{dy}{dx} = 0$$ ½ m

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0 \quad \text{[Using (i)]}$$ 1 m

20. Given differential equation can be written as

$$\left[2x - x\log\left(\frac{y}{x}\right)\right]\frac{dy}{dx} = y \ \ or \ \ \frac{dy}{dx} = \frac{y}{2x - x\log\left(y/x\right)}$$

$$= \frac{y/x}{2-\log\left(y/x\right)} = f\left(y/x\right)$$

hence, the differential equation is homogeneous. 1 m

Taking $\frac{y}{x} = v$ or $y = vx$ to get $\frac{dy}{dx} = v + x\frac{dy}{dx}$ 1 m

$\therefore \quad v + x\frac{dv}{dx} = \frac{v}{2-\log v} \Rightarrow x\frac{dv}{dx} = \frac{v-2v+v\log v}{2-\log v} = \frac{v\log - v}{2-\log v}$

$\Rightarrow \int \frac{2-\log v}{v(\log v-1)}dv = \int \frac{dx}{x} \Rightarrow \int \frac{\log v-1-1}{v(\log v-1)}dv = -\int \frac{dx}{x}$ 1 m

$\Rightarrow \quad \log v - \log|\log v - 1| + \log x = \log c$

$\Rightarrow \frac{vx}{\log v-1} = c \text{ or } y = c\left(\log {}^{y}\!/_{x} - 1\right)$ 1 m

21. $y = \cos^{-1}\left[\frac{3}{5}x + \frac{4}{5}\sqrt{1-x^2}\right]$

$= \cos^{-1}\left[\frac{3}{5}\cdot\cos\theta + \frac{4}{5}\sin\theta\right]$ where $x = \cos\theta$ 1 m

$= \cos^{-1}[\cos\alpha\cdot\cos\theta + \sin\alpha\cdot\sin\theta]$, $\because$ if $\frac{3}{5} = \cos\alpha$, then $\frac{4}{5} = \sin\alpha$ 1 m

$= \cos^{-1}[\cos(\alpha-\theta)] = \alpha-\theta = \cos^{-1}\left({}^{3}\!/_{5}\right) - \cos^{-1}x$ 1 m

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\left[\text{Note : Answer can be } -\frac{1}{\sqrt{1-x^2}}\right]$ 1 m

22. $I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx = 5\int_1^2 1 - \frac{4x+3}{x^2+4x+3}dx$ 1 m

$= 5[x]_1^2 - 10\int_1^2 \frac{2x+4-\frac{5}{2}}{x^2+4x+3}dx$ ½ m

$$= 5 - 10\left[\log\left|x^2+4x+3\right|\right]_1^2 + 25\int_1^2 \frac{1}{(x+2)^2-(1)^2}\,dx$$ 1 m

$$= 5 - 10\ \log\frac{15}{8} + 25\cdot\frac{1}{2}\left[\log\left|\frac{x+2-1}{x+2+1}\right|\right]_1^2$$ 1 m

$$= 5 - 10\ \log\frac{15}{8} + \frac{25}{2}\ \log\frac{6}{5}$$ ½ m

Note: If solved using partial fractions, the answer may be of the form

$$5 + \frac{5}{2}\ \log\frac{3}{2} - \frac{45}{2}\ \log\left(\frac{5}{4}\right)$$

SECTION - C

23. Getting (x_1, y_1) at $\theta = \pi/4 = \left(\frac{\sqrt{2}-1}{\sqrt{2}}, \frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)$ 1 m

$$\frac{dy}{dx} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \left(\sqrt{2}-1\right)$$ 1 m

slope of tangent $= \left(\sqrt{2}-1\right)$ and ½ m

slope of Normal $= -\frac{1}{\sqrt{2}-1}$ ½ m

Equation of tangent is

$$y - \left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) = \left(\sqrt{2}-1\right)\left[x - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\right]$$ 1½ m

Equation of Normal is

$$y - \left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}-1}\left[x - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\right]$$ 1½ m

24. Equation of plane containing the given line and passing throngh P(1, 1, 1) is

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1+3 & 1-1 & 1-5 \\ 3 & -1 & -5 \end{vmatrix} = 0$$ 2 m

$$\Rightarrow \quad -4(x-1) + 8(y-1) - 4(z-1) = 0$$

$$x - 2y + z = 0 \quad \text{.................................(i)}$$ 2 m

Since (– 1, 2, 5) lies on (i) [– 1 – 4 + 5 = 0] and 1 m

$$\left(\hat{i} - 2\hat{j} - 5\hat{k}\right) \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 1 + 4 - 5 = 0$$

1 m

$\therefore$ The line $\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} - 5\hat{k}\right)$ lies on the plane

25. LHS $= \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + p\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$ 1 m

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$ ½ m

$$= (1 + pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$ ½ m

$$= (1+pxyz)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$ 1 m

$$= (1+pxyz)(x-y)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & -1 & -(x+y) \\ 0 & 1 & z+x \end{vmatrix}$$ ½ m

$$= (1+pxyz)(x-y)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 0 & z-y \\ 0 & 1 & z+x \end{vmatrix} \quad R_2 \to R_2 + R_3$$ 1 m

$$= (1+pxyz)(x-y)(y-z)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 0 & -1 \\ 0 & 1 & z+x \end{vmatrix}$$ ½ m

$$= (1+pxyz)(x-y)(y-z)(z-x)\cdot 1 = \text{R.H.S.}$$ 1 m

OR

Writing $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_2 \to R_2 + R_1 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A$ 1 m

$$R_3 \to R_3 + 2R_2 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ 1 m

$$R_1 \to R_1 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 10 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ ½ m

$$R_1 \to R_1 - 2R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ ½ m

$$\text{Hence } A^{-1} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$ 1 m

26. E_1 : Bag contains 2 white balls and 2 non whites

E_2 : Bag contains 3 white balls and 1 non white 1 m

E_3 : Bag contains 4 white balls

A : Getting two white balls

$$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{3}$$ ½ m

$$P(A/E_1) = \frac{2c_2}{4c_2} = \frac{1}{6},\ P(A/E_2) = \frac{3c_2}{4c_2} = \frac{1}{2},\ P(A/E_3) = 1$$ 1½ m

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) + P(E_3)\ P(A/E_3)}$$ 1 m

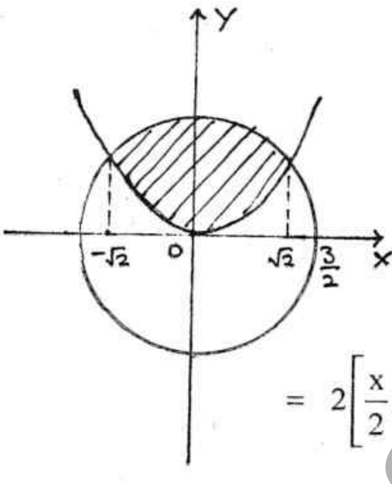
$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot 1}$$ 1 m

$$= \frac{6}{10} = \frac{3}{5}$$ 1 m

27. Correct Figure 1 m

Solving $4x^2+4y^2=9$ and $x^2 = 4y$

We get $x = \pm\sqrt{2}$ (as points of intersection)



or $y = \frac{1}{2}$ ½ m

Required area

$$= 2\left[\int_0^{\sqrt{2}}\sqrt{\frac{9}{4}-x^2}\,dx - \int_0^{\sqrt{2}}\frac{1}{4}x^2\,dx\right]$$ 2 m

$$= 2\left[\frac{x}{2}\sqrt{\frac{9}{4}-x^2}+\frac{9}{8}\sin^{-1}\frac{2x}{3}-\frac{x^3}{12}\right]_0^{\sqrt{2}}$$ 1½ m

$$= 2\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}+\frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}-\frac{2\sqrt{2}}{12}\right)$$ ½ m

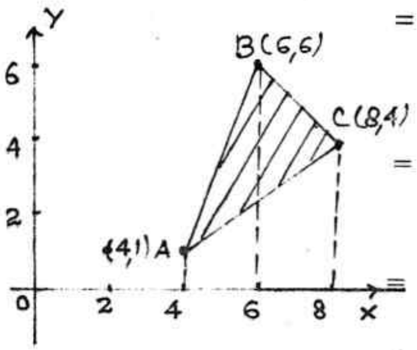
$$= \left(\frac{\sqrt{2}}{6}+\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right) \text{ sq. units}$$ ½ m

OR

Equations of AB, BC and AC respectively are

$$y = \frac{5}{2}x - 9,\ \ y = 12 - x,\ \ y = \frac{3}{4}x - 2$$ 1½ m

Required area



$$= \int_4^6 \left(\frac{5}{2}x - 9\right) dx + \int_6^8 (12 - x)\, dx - \int_4^8 \left(\frac{3}{4}x - 2\right) dx$$ 2 m

$$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$ 1½ m

$= (7 + 10 - 10)$ sq units 1 m

$= 7$ sq units

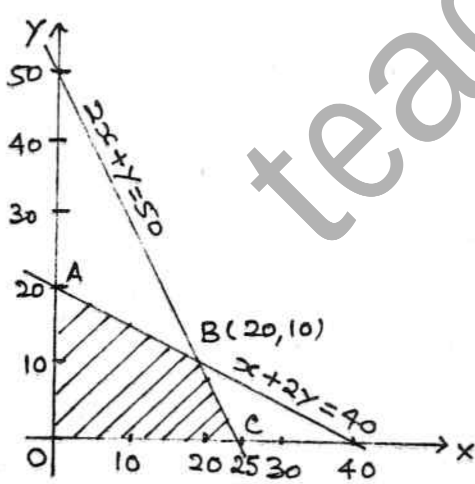
28. Let x cakes of first type and y cakes of second type are made

Maximise $S = x + y$ 1 m

subject to $300x + 150y \leq 7500$ or $2x + y \leq 50$

$15x + 30y \leq 600$ or $x + 2y \leq 40$ 2 m

$x \geq 0, y \geq 0$



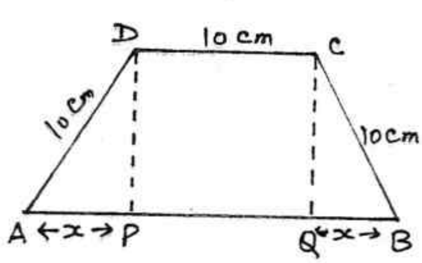
Correct graph 2 m

Vertices of feasible region are

A (0, 20), B (20, 10) C (25, 0)

Maximum cakes $= 20 + 10 = 30$ 1 m

29.



Let ABCD be the given trapezium

with AD = DC = BC = 10 cm. 1 m

Draw DP $\perp$ AB and CQ $\perp$ AB

and let AP = x cm => QB = x cm

$\therefore$ Area of trapezium, $A = \frac{1}{2}\left[10+(10+2x)\right]\sqrt{100-x^2}$

1 m

$$A = (x+10)\sqrt{100-x^2}$$

Let $S = (x+10)^2 . (100-x^2) \Rightarrow \frac{ds}{dx} = -2x(x+10)^2 + 2(x+10)(100-x^2)$

$= 2(x+10)^2 \cdot (-x+10-x)$ 1 m

$= 2(x+10)^2 \cdot (10-2x)$

$\frac{ds}{dx} = 0 \Rightarrow x = 5$ [rejecting x = - 10] 1 m

$\frac{d^2s}{dx^2} = -4(x+10)^2 + 4(x+10)(10-2x) = -900$ (– ve) 1 m

$\therefore$ Maximum Area $A = 15\sqrt{75}\ cm^2$ *or* $75\sqrt{3}\ cm^2$ 1 m

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. zero 2. $x = 4$ 3. $\frac{1}{3}$ 4. 49 5. $-\frac{1}{4}\tan(7-4x)+c$ — 1x10 = 10 m

6. $\frac{\pi}{5}$ 7. $\frac{\pi}{3}$ 8. –3 9. x 10. $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$

SECTION - B

11. $\forall$ a, b $\in$ N, (a, b) S (a, b) $\because$ a + b = b + a. Hence S is reflexive — 1 m

a, b, c, d $\in$ N, Let (a, b) S (c, d) $\therefore$ a + d = b + c

$\Rightarrow$ c + b = d + a $\therefore$ (c, d) S (a, b) $\Rightarrow$ S is symmetric — 1 m

a, b, c, d, e, f $\in$ N, Let (a, b) S (c, d) and (c, d) S (e, f)

$\therefore$ a + d = b + c and c + f = d + e

adding to get a + f = b + c $\Rightarrow$ (a, b) S (e, f) $\therefore$ S is transitive — 1½ m

Hence S is an equivalence relation — ½ m

12. LHS $= \tan^{-1}\left[\dfrac{x + \dfrac{2x}{1-x^2}}{1 - x\,\dfrac{2x}{1-x^2}}\right]$ — 2 m

$= \tan^{-1}\left[\dfrac{x(1-x^2)+2x}{1-x^2-2x^2}\right]$ — 1 m

$= \tan^{-1}\left[\dfrac{3x-x^3}{1-3x^2}\right]$ = RHS. — 1 m

OR

LHS $= \cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

$$= \cos\left[\tan^{-1}\left\{\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}\right]$$ 1 m

$$= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right] = \cos\left[\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right]$$ 1+1 m

$$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}} = \text{R.H.S}$$ 1 m

13. Let event A is that the family has two boys

(i) event B: At least one is a boy

P(both boys, given that at least one is a boy) = P(A/B) ½ m

$$= \frac{P(A \cap B)}{P(B)} = \frac{P\{(B, B)\}}{P\{(B, G), (G, B), (B, B),\}}$$ ½ + ½ m

$$= \frac{1/4}{3/4} = \frac{1}{3}$$ ½ m

(ii) event C: the elder child is a boy

P(both boys, given that at elder child is a boy) = P(A/C)

$$= \frac{P(A \cap C)}{P(C)} = \frac{P\{(B, B)\}}{P\{(B, G), (B, B),\}}$$ 1 m

$$= \frac{1/4}{2/4} = \frac{1}{2}$$ 1 m

14. Any point Q on the given line is Q $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ 1 m

$PQ^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 17\lambda^2 - 18\lambda - 16\lambda + 25$ 1 m

$PQ^2 = (5)^2 \Rightarrow 17\lambda(\lambda - 2) = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = 2$ 1 m

$\therefore$ The points are Q (–2, –1, 3) and R (4, 3, 7) ½+½ m

OR

Normal to the plane passing through A, B and C

$$\text{is} \quad \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 12\hat{i} - 16\hat{j} + 12\hat{k} \text{ or } 3\hat{i} - 4\hat{j} + 3\hat{k}$$ 1½ m

$\therefore$ Equation of plane is $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$ or $3x - 4y + 3z - 19 = 0$ 1½ m

$$\text{Distance of P(6, 5, 9) from the plane} = \frac{|18 - 20 + 27 - 19|}{\sqrt{(3)^2 + (-4)^2 + (3)^2}}$$

$$= \frac{6}{\sqrt{34}}$$ 1 m

15. $$AB = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} (-1, 2, 1) = \begin{pmatrix} -1 & 2 & -1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{pmatrix}$$ 1½ m

$$\therefore \text{ LHS} = (AB)' = \begin{pmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ -1 & -4 & 3 \end{pmatrix} \text{ .... (i)}$$ ½ m

$$B' = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \quad A' = (1, -4, 3)$$ ½+½ m

$$\text{RHS} = B'A' = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} (1 \ -4 \ 3) = \begin{pmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ -1 & -4 & 3 \end{pmatrix} = \text{LHS}$$ 1 m

16. $2\vec{a}-\vec{b}+3\vec{c} = (2\hat{i}+2\hat{j}+2\hat{k})-(4\hat{i}-2\hat{j}+3\hat{k})+(3\hat{i}-6\hat{j}+3\hat{k}) = \hat{i}-2\hat{j}+2\hat{k}$ 2 m

$|2\vec{a}-\vec{b}+3\vec{c}| = 3$ 1 m

$\therefore$ Required vector $= 2\hat{i}-4\hat{j}+4\hat{k}$ 1 m

OR

A vector perpendicutar to $\vec{a}$ and $\vec{b} = \vec{a}\times\vec{b} = 32\hat{i}-\hat{j}-14\hat{k}$ ½+1 m

Let $\vec{d} = \lambda(32\hat{i}-\hat{j}-14\hat{k})$ ½ m

$\therefore\ \vec{c}\cdot\vec{d} = 18 \Rightarrow \lambda(64+1-56) = 18 \Rightarrow \lambda = 2$ 1½ m

$\therefore\ \vec{d} = 64\hat{i}-2\hat{j}-28\hat{k}$ ½ m

17. Writing as $\frac{dy}{dx}+\frac{2x}{x^2+1}\cdot y = \frac{\sqrt{x^2+4}}{x^2+1}$ 1 m

$\int\frac{2x}{x^2+1}dx = \log|x^2+1|$ $\therefore$ Integrating factor $= (x^2+1)$ 1 m

$\therefore$ Solution is $y\cdot(x^2+1) = \int\sqrt{x^2+4}\,dx$ 1 m

$\Rightarrow\ y\cdot(x^2+1) = \frac{x}{2}\cdot\sqrt{x^2+4} + 2\log\left|x+\sqrt{x^2+4}\right| + c$ 1 m

OR

Writing as $dy = \frac{2x^2+x}{(x+1)(x^2+1)}dx$ ½ m

$\Rightarrow\ y = \int\left(\frac{\frac{1}{2}}{x+1}+\frac{\left(\frac{3}{2}x-\frac{1}{2}\right)}{x^2+1}\right)dx$ 1½ m

$$y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + c$$ 2 m

18. $y = \cos^{-1}\left[\frac{3}{5}x + \frac{4}{5}\sqrt{1-x^2}\right]$

$= \cos^{-1}\left[\frac{3}{5}\cdot\cos\theta + \frac{4}{5}\sin\theta\right]$ where $x = \cos\theta$ 1 m

$= \cos^{-1}[\cos\alpha\cdot\cos\theta + \sin\alpha\cdot\sin\theta]$, $\because$ if $\frac{3}{5} = \cos\alpha$, then $\frac{4}{5} = \sin\alpha$ 1 m

$= \cos^{-1}[\cos(\alpha-\theta)] = \alpha-\theta = \cos^{-1}\left(3/5\right) - \cos^{-1}x$ 1 m

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\left[\text{Note : Answer can be } -\frac{1}{\sqrt{1-x^2}}\right]$ 1 m

19. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+2\,y/x}{1-y/x} = f\left(y/x\right)$$

hence, the differential equation is homogeneous. 1 m

Taking $\frac{y}{x} = v$ or $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$ ½ m

$\therefore\ v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$ or $x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+v+v^2}{1-v}$

$\Rightarrow \int\frac{v-1}{v^2+v+1}dv = -\int\frac{dx}{x}$ 1 m

$$\Rightarrow \frac{1}{2}\int \frac{2v+1-3}{v^2+v+1}dv = -\log|x|+c \;\; or \;\; \frac{1}{2}\log|v^2+v+1| - \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= -\log|x|+c$$

$$\Rightarrow \log|v^2+v+1| + \log x^2 = 2\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)+c$$ 1 m

$$\Rightarrow \log|y^2+xy+x^2| = 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)+c$$ ½ m

20. $y = \operatorname{cosec}^{-1} x \Rightarrow \frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}}$ 1 m

$$\Rightarrow x\sqrt{x^2-1}\cdot\frac{dy}{dx} = -1$$ ½ m

$$\Rightarrow x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\left(x\frac{2x}{2\sqrt{x^2-1}}+\sqrt{x^2-1}\right)=0$$ 1½ m

$$\Rightarrow x(x^2-1)\frac{d^2y}{dx^2}+\frac{dy}{dx}(2x^2-1)=0$$ 1 m

21. $I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx = 5\int_1^2 1-\frac{4x+3}{x^2+4x+3}dx$ 1 m

$$= 5[x]_1^2 - 10\int_1^2 \frac{2x+4-\frac{5}{2}}{x^2+4x+3}dx$$ ½ m

$$= 5-10\left[\log|x^2+4x+3|\right]_1^2 + 25\int_1^2 \frac{1}{(x+2)^2-(1)^2}dx$$ 1 m

$$= 5 - 10\ \log\frac{15}{8} + 25\cdot\frac{1}{2}\left[\log\left|\frac{x+2-1}{x+2+1}\right|\right]_1^2$$ 1 m

$$= 5 - 10\ \log\frac{15}{8} + \frac{25}{2}\ \log\frac{6}{5}$$ ½ m

Note: If solved using partial fractions, the answer may be of the form

$$5 + \frac{5}{2}\ \log\frac{3}{2} - \frac{45}{2}\ \log\left(\frac{5}{4}\right)$$

22. Here $I = \int \frac{x+2}{\sqrt{x^2-5x+6}}\,dx = \frac{1}{2}\int\frac{2x-5+9}{\sqrt{x^2-5x+6}}\,dx$ 1 m

$$= \frac{1}{2}\int\frac{2x-5}{\sqrt{x^2-5x+6}}\,dx + \frac{9}{2}\int\frac{1}{\sqrt{\left(x-\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\,dx$$ 1 m

$$= \sqrt{x^2-5x+6} + \frac{9}{2}\ \log\left|\left(x-\frac{5}{2}\right)+\sqrt{x^2-5x+6}\right| + c$$ 1+1 m

SECTION - C

23. Writing given system as $\begin{pmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-4\\ 2\\ 11\end{pmatrix}$ or $AX = B$ ½ m

$|A| = 1(-6) - 2(-14) - 3(-15) = 67 \neq 0\ \therefore\ \times = A^{-1}.B.$ 1+½ m

$a_{11} = -6,\quad a_{12} = 14,\quad a_{13} = -15$
$a_{21} = 17,\quad a_{22} = 5,\quad a_{23} = 9$ [1 m. for every 4 correct co-factors] 2 m
$a_{31} = 31,\quad a_{32} = -8,\quad a_{33} = -1$

$$\Rightarrow A^{-1} = \frac{1}{67}\begin{pmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{pmatrix}$$ ½ m

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{67}\begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}\begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$\therefore\ x = 3,\ y = -2,\ z = 1$ 1½ m

OR

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} \text{ Using } c_1 \to c_1+c_2+c_3$$ 1 m

$$= (a+b+c)\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix} \text{ Using } R_1 \to R_1-R_2,\ R_2 \to R_2-R_3$$ 2 m

$= (a+b+c)\ [(b-c)(a-b)-(c-a)^2]$ 1 m

$= (a+b+c)\ [ab+bc+ca-a^2-b^2-c^2]$ 1 m

$= -\frac{1}{2}(a+b+c)\ [(a-b)^2+(b-c)^2+(c-a)^2]$

1 m

Which is negative for a, b, c positive and unequal

24. Correct Figure 1 m

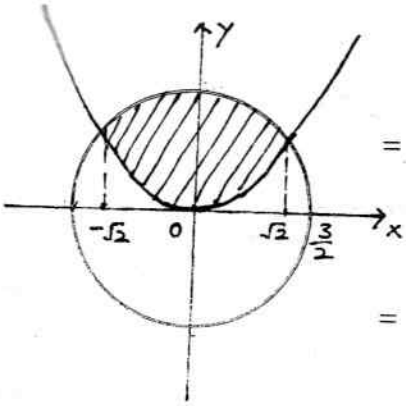
Solving $4x^2+4y^2=9$ and $x^2 = 4y$

We get $x = \pm\sqrt{2}$ (as points of intersection)

or $y = \frac{1}{2}$ ½ m

Required area

$$= 2\left[\int_0^{\sqrt{2}} \sqrt{\frac{9}{4} - x^2}\, dx - \int_0^{\sqrt{2}} \frac{1}{4} x^2\, dx\right]$$ 2 m

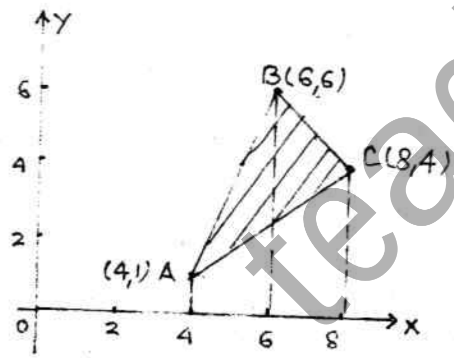


$$= 2\left[\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3} - \frac{x^3}{12}\right]_0^{\sqrt{2}}$$ 1½ m

$$= 2\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{2\sqrt{2}}{12}\right)$$ ½ m

$$= \left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right) \text{sq. units}$$ ½ m

OR



Equations of AB, BC and AC respectively are

$$y = \frac{5}{2}x - 9,\ \ y = 12 - x,\ \ y = \frac{3}{4}x - 2$$ 1½ m

Required area

$$= \int_4^6 \left(\frac{5}{2}x - 9\right) dx + \int_6^8 (12 - x)\, dx - \int_4^8 \left(\frac{3}{4}x - 2\right) dx$$ 2 m

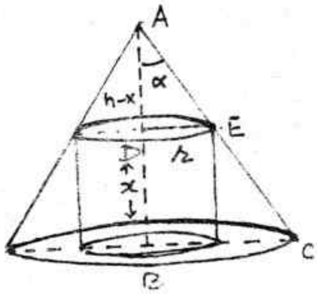
$$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$ 1½ m

$= (7 + 10 - 10)$ sq units

1 m

$= 7$ sq units

25. Correct Figure 1 m



Let radius of cylinder be r and height x.

$\therefore$ Volume = $\pi\ r^2 x$ ....(i) 1 m

From figure $\frac{r}{h-x}$ = tan α or r = (h − x) tan α

$\therefore$ Volume (V) = $\pi x (h-x)^2 \tan^2\alpha$

$= \pi \tan^2\alpha \left[h^2x - 2hx^2 + x^3\right]$ 1 m

$\frac{dv}{dx} = \pi \tan^2\alpha \left[h^2 - 4hx + 3x^2\right]$ ½ m

$= \pi \tan^2\alpha \left[(3x-h)(x-h)\right]$

$\frac{dv}{dx} = 0 \Rightarrow x = \frac{h}{3}$ (rejecting x = h) ½ m

$\frac{d^2v}{dx^2} = \pi \tan^2\alpha \left[-4h + 6x\right] = \pi \tan^2\alpha \left[-4h + 6h\right] = -ve$ 1 m

$\therefore$ Maximum volume $= \pi \tan^2\alpha \left[\frac{h^3}{3} - 2\frac{h^3}{9} + \frac{h^3}{27}\right]$

1 m

$= \frac{4}{27} = \pi h^3 \tan^2\alpha$

26. Full marks to be given to every candidate for this question. 6 m

27. E_1 : Bag contains 2 white balls and 2 non whites

E_2 : Bag contains 3 white balls and 1 non whites 1 m

E_3 : Bag contains 4 white balls

A : Getting two white balls

$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{3}$ ½ m

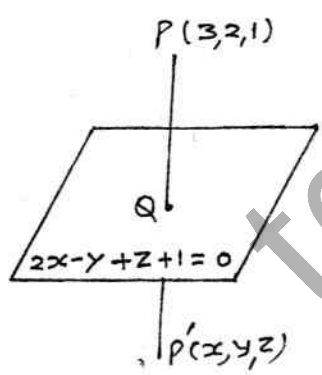
$P(A/E_1) = \frac{2c_2}{4c_2} = \frac{1}{6},\ P(A/E_2) = \frac{3c_2}{4c_2} = \frac{1}{2},\ P(A/E_3) = 1$ 1½ m

$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) + P(E_3)\ P(A/E_3)}$ 1 m

$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot 1}$ 1 m

$= \frac{6}{10} = \frac{3}{5}$ 1 m

28. Let Q be the foot of perpendicular from P to the plane



$\therefore$ Equation of PQ is $\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ 1 m

Any point on this line is $(2\lambda+3, -\lambda+2, \lambda+1)$ ½ m

If this point is Q, then it must satisfy the equation of plane

$\therefore\ 2(2\lambda+3) - (-\lambda+2) + (\lambda+1) + 1 = 0$ 1 m

$\Rightarrow\ \lambda = -1$ ½ m

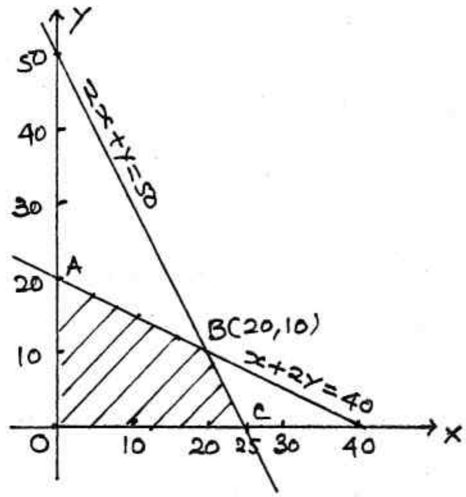
$\therefore$ coordinates of foot of perpendiculare are Q (1, 3, 0) ½ m

Perpendicular distance = PQ = $\sqrt{4+1+1} = \sqrt{6}$ units 1 m

Let $P'(x, y, z)$ be the image, then $\frac{x+3}{2} = 1, \frac{y+2}{2} = 3, \frac{z+1}{2} = 0$ ½ m

$\therefore\ P'$ is $(-1, 4, -1)$ 1 m

29. Let x cakes of first type and y cakes of second type are made



Maximise $S = x + y$ 1 m

subject to $300x + 150y \le 7500$ or $2x + y \le 50$

$15x + 30y \le 600$ or $x + 2y \le 40$ 2 m

$x \ge 0, y \ge 0$

Correct graph 2 m

Vertices of feasible region are

A (0, 20), B (20, 10) C (25, 0)

Maximum cakes $= 20 + 10 = 30$ 1 m